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Using Rayleigh waves to extract the shear wave velocity in the soil deposits.

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Abstract:

The application of using Rayleigh (Surface) waves to extract the shear wave velocity in soil and rock formation have increased recently in seismic geotechnical engineering as well as environmental engineering. The shear wave velocity (Vs), is a soil mechanical property that can be advantageously measure in both the field and laboratory under real and controlled conditions.

In this study, the modeling of the surface wave data was performed using the MATLAB software. The parameters use in this work for modelling are taking from reference [1].

The modeling of the surface wave data results a shear wave velocity (Vs) of 100-700m/s covering the top soil to weathering and up to bedrock corresponding to a depth range of 2-10m. The Phase velocity vs Frequency, the shear wave velocity vs depth, and the vertical particle displacement profile vs penetration depth is analyzed.

Keyword: Rayleigh waves, shear waves velocity and soil deposit

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1.0 General Introduction

Here, the shear wave velocity of a soil is modelled using MATLAB. The outcomes from the program result in the variations of Shear wave velocity versus depth. The soil parameters were used from [1] to do the simulation.

The Shear wave velocity is dependent on the Special Penetration Test value which is used in soil mechanics as well as foundation engineering and is the index value for formation hardness.

A Rayleigh wave is a surface wave which travels along a free surface like the interface between earth and air. The Rayleigh wave is produced by the interference of shear velocity and P- waves. The fundamental mode for the Rayleigh wave (which moves from left to right) shows a particle motion which has elliptical shape and direction as counter clockwise. Only the vertical plane contains the motion. This is also consistent to the direction for the wave propagation. A larger phase velocity is shown by larger wavelengths for a mode provided (which have larger sensitivity for elastic properties for layers which are deep). The smaller wavelengths show sensitivity for the physical properties for the layers of surface. A mode for the surface wave has a particular phase velocity corresponding to a wavelength. Due to this, dispersion is observed in the seismic signal. The dispersive phase velocity for the surface wave (which is Rayleigh) can be inverted to find the Shear wave velocity.

1.1 Problem description

The idea behind using this method is to extract the variations of Shear wave velocity values inside the soil and rock formations.

Based on the theory of elasticity, it is directly related to the small strain stiffness of the geomaterials which has main contribution in many geotechnical problems and foundation designing.

It is proved by large number of studies that using this technique make it possible to calculate the required elastic properties for the actual applications in a cost effective and reliable manner.

2.0 Literature Review

The Rayleigh waves have been used to extract the shear wave velocity in the soil deposit. The elastic properties for the near – surface materials as well as the manner in which they affect the seismic wave propagation are significant while studying the environmental engineering [1]. The construction engineering includes an important parameter, i.e., Share wave velocity. The Shear wave velocity is dependent on the Special Penetrated value which is used in soil mechanics as well as foundation engineering and is the index value for formation hardness.

The surface wave is dispersive and guided in nature. A Rayleigh wave is a surface wave which travels along a free surface like the interface between earth and air. The Rayleigh wave is produced by the interference of Shear velocity and P waves. The fundamental mode for the Rayleigh wave (which moves from left to right) shows a particle motion which has elliptical shape and direction as counter clockwise. Only the vertical plane contains the motion. This is also consistent to the direction for the wave propagation [2]. A larger phase velocity is shown by larger wavelengths for a mode provided (which have larger sensitivity for elastic properties for layers which are deep). The smaller wavelengths show sensitivity for the physical properties for the layers of surface. A mode for the surface wave has a particular phase velocity corresponding to a wavelength. Due to this, dispersion is observed in the seismic signal [3]. The dispersive phase velocity for the surface wave (which is Rayleigh) can be inverted to find the Shear wave velocity. In the solid homogeneous half – space, the Rayleigh wave has a velocity of 0.9194 v and it isn't dispersive. Here, the Poisson's ratio is 0.25 and 'v' refers to Shear wave velocity for the half – space.

The phase velocity for the Rayleigh wave in case of the layer earth model depends on the frequency. The properties of earth are divided into 4 parts. They are: Shear wave velocity, P – wave velocity, thickness and density of layers. The Jacobian matrix can be analyzed to give a measurement of the sensitivity of the dispersion – curve to the properties of the earth. In the higher frequency range, with frequency > 5 Hz, the major factors include the Shear wave velocity and thickness of the layer [4]. The Levenberg – Marquardt as well as singular value decomposition methods can be used in the higher frequency range for an iteration - based solution using the weighted equation. In the Levenberg – Marquardt method, the value of damping factor is chosen to ensure that the weighted solution converges.

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3.0 Inversion Algorithm

Shear wave velocities can be inverted adequately from Rayleigh waves phase velocities.

Shear wave velocities (earth model parameters) can be represented as the elements of a vector x of length n, or $x = [vs_{1,} vs_2 \, , v_{s3} \ldots v_{sn}]^T$.

Similarly, the measurements (data) of Rayleigh -wave velocities at m, different frequencies can be represented as the elements of a vector b of length m or b= $[b_1, b_2, b_3, \ldots . b_n]^T$

We can employ the matrix theory, that is, J$\Delta x = \Delta b$, where Δx is a modification of the initial estimation, and J is the Jacobian matrix with m rows and n column (m>n).

Marquardt (1963) points out that the damping factors controls the direction of Δx and the speed Of convergence. The damping factor also acts as a constraint on the model space (Tarantola, 1987, chapter 4). By adjusting the damping factor, we can improve processing speed and guarantee the stable convergence of the inversion. The Jacobian matrix equation also suggests phase velocity data as a function of frequency possess different resolving powers for determining Shear wave velocities at different depth

4.0 Advantages and limitations of using Rayleigh waves to extract the shear wave velocity in the soil deposits.

This method is non – destructive without need to drilling, which means, it can be done

quickly and is very cost effective. This method can measure the share wave velocity in ground under the natural conditions.

Limitations: However, depending on the field conditions, this technique may not work well in some cases, but works very well in most cases.

5.0 Methodology

Introduction:

The application of using Rayleigh waves to extract the shear wave velocity in soil

and rock formation have increased recently in Geo-technical engineering as well as Environmental engineering.

In order to produce useful results from the problem stated above, the project will be undertaken

Through the procedures below:

1. An extensive review of literature on the subject on the subject matter, couple with an intensive desk study on various laboratory test data that are available.

2. Review the various specifications that have been developed by some authors. This project provides, conclusion, recommendations, by the use of the literature review, desk study, experiment data analysis, performance analysis and model analysis.

6.0 Rayleigh wave equations

Let us consider an isotropic, homogeneous and linearly elastic medium which fills the half-space defined by y ≥ 0. We work in a Cartesian coordinate system PXYZ. The equation describing the displacement vector U of an infinitesimal element in a medium as function of time is a linear electrodynamic equation (we neglect in this equation the volume forces which are not important in this context):

\begin{equation}

\[ρ\frac{∂^2U}{∂t^2}-μ ΔU−(λ+μ) ∇(div U = 0\]   (2.1)

\end{equation}

\[U=U_{l}+U_{t}=∇\phi+\vec{rot} \psi\]

\end{equation}   (2.2)

With $U_{l}=∇\phi=\vec{grad}\phi$, $U_{t}=\vec{rot}\psi$, and ϕ, ψ are defined as the scalar and the vector potentials respectively

Substituting (2.2) into (2.1) leads to the two independent equations

\begin{equation}

\[ρ\frac{∂^2U}{∂t^2}-(λ+2μ)ΔU=0\]   (2.3)

\end{equation}

\begin{equation}

\[ρ\frac{∂^2U}{∂t^2}-μ ΔU_{t} = 0\]   (2.4)

\end{equation}

The equation (2.3) describes the propagation of longitudinal waves, and (2.4) describes the propagation of transversal waves.

The Rayleigh wave propagates in the positive direction along the border of the

half-space represented by the x-axis. Writing the explicit form of $U_{l,t}$ we find that the potentials ϕ, ψ are solutions of the ordinary wave equations

\begin{equation}

\[\frac{∂^2\phi}{∂x^2}+\frac{∂^2\phi}{∂y^2}-c_{t}^2\phi=0;\] (2.5)

\end{equation}

\begin{equation}

\[\frac{∂^2\psi}{∂x^2}+\frac{∂^2\psi}{∂y^2}-c_{t}^2\psi=0;\]   (2.6)

\end{equation}

where c_{l} and c_{t} are wave numbers corresponding to longitudinal and transversal waves, defined by

$c_{l}=\sqrt[w]{\frac{\rho}{λ + 2μ}}$, $c_{l}=\sqrt[w]{\frac{\rho}{μ}}$.

We look for solutions propagating in the $x-axis$ direction and with amplitude depending only on y:

$\phi(x,y,t)=F(y)e^{i(cx−ωt)}$, $\psi(x,y,t)=G(y)e^{i(cx−ωt)}$

with c representing the phase velocities on the surface. Substituting the explicit expressions of \phi and \psi into (2.5) and (2.6), we get

\begin{equation}

\[\frac{∂^2F(y)}{∂y^2}=(c^2-c_{l}^2)F(y)\], (2.7)

\end{equation}

\begin{equation}

\[\frac{∂^2G(y)}{∂y^2}=(c^2-c_{t}^2)G(y)\], (2.8)

\end{equation}

The above equations for $F(y)$, $G(y)$ are linear differential equations. We will take the negative exponent solution, which is the physical solution in opposite the positive one which assume that the wave is increasing exponential in function of y - and this cannot be a realistic situation,

$\phi(x,y,t)=A_{l}e^[-\sqrt{(c^2-c_{l}^2)(y)}+i(cx−ωt)]$,

$\psi(x,y,t)=A_{t}e^[-\sqrt{(c^2-c_{t}^2)(y)}+i(cx−ωt)]$.

With A_{l}, A_{t} At arbitrary constants, we assume that $c^2>c_{l}^2)$ and $c^2>c_{t}^2)$ (or $c^2>c_{t}^2)>c_{l}^2)$),and $i^2 = −1$.

From (2.2) we obtain

$U=U_{l}+U_{t}=\vec{grad}\phi+∇\psi=(\frac{∂\phi}{∂x}-\frac{∂\psi}{∂y})e_{x}+(\frac{∂\phi}{∂y}-\frac{∂\psi}{∂x})e_{y}$, where e_{x}, e_{y} are based vectors. The displacement components U_{x}, U_{y} are given by

$U_{x}=\frac{∂\phi}{∂x}-\frac{∂\psi}{∂y}$, $U_{y}=\frac{∂\phi}{∂y}+\frac{∂\psi}{∂x})$

Using Hook's law in elastic solids:

\begin{equation}

$σ_{ij}=a_{ijkl} \in_{kl}$ (2.9)

\end{equation}

The exact expressions for the roots of Rayleigh wave equation the stress components $σ_{xx}$, $σ_{yy}$, $σ_{xy}$ are given by

$σ_{xx}=λ(\frac{∂^2\phi}{∂x^2}+\frac{∂^2\phi}{∂y^2})+2u(\frac{∂^2\phi}{∂x^2}-\frac{∂^2\psi}{∂x∂y})$

$=(λ+2u)(\frac{∂^2\phi}{∂x^2}+\frac{∂^2\phi}{∂y^2})-2u(\frac{∂^2\phi}{∂x^2}-\frac{∂^2\psi}{∂x∂y})$

$σ_{yy}=λ(\frac{∂^2\phi}{∂x^2}+\frac{∂^2\phi}{∂y^2})+2u(\frac{∂^2\phi}{∂x^2}+\frac{∂^2\psi}{∂x∂y})$

$=(λ+2u)(\frac{∂^2\phi}{∂x^2}+\frac{∂^2\phi}{∂y^2})-2u(\frac{∂^2\phi}{∂x^2}-\frac{∂^2\psi}{∂x∂y})$

$σ_{xy}=u(2\frac{∂^2\phi}{∂x∂y}+\frac{∂^2\phi}{∂x^2}-\frac{∂^2\psi}{∂y^2})$

Using the boundary conditions

$σ_{yy}(x,y=0,z,t)=σ_{xy}(x,y=0,z,t)=0$,

and substituting the expressions of ϕ and ψ in these conditions we get the following

system containing the arbitrary constants A_{l} and A_{t}:

\begin{equation}

_{2ic\sqrt{c^2-c_{l}^2}A_{l}+(2c^2-c_{t}^2)A_{t}=0)} (2.10)

\{c^2\frac{λ}{2μ}A_{l}(c^2-c_{l}^2)(1+\frac{λ}{2μ}A_{l}+ic\sqrt{c^2-c_{t}^2}A_{t}=0),

\end{equation}

The non-trivial solutions lead to the condition:

\begin{equation}

4c^2\sqrt{c^2-c_{l}}{\sqrt{c^2-c_{t}}-(c^2-c_{t}^2)^2=0} (2.11)

\end{equation}

The polynomial form of the above equation is

\begin{equation}

η^6−8η^4+8(3−2ξ^2)η^2−16((1−ξ^2)=0   (2.12)

\end{equation}

The above equation is called the Rayleigh wave equation, where $η =\frac{c_{t}}{c}$ and $ξ=\frac{c_{l}}{c_{t}}$and c, c_{l}, c_{t} are defined as the wave numbers for phase velocities of surface, longitudinal and transversal waves respectively.

The velocity c at which Rayleigh waves propagate over an isotropic and elastic surface defined on the half-space $y ≥ 0$ is the root of the equation (2.12). After this change of variables, we have

$η^2=θ=(\frac{c_{t}}{c})^2$, $ξ^2=α=(\frac{c_{l}}{c_{t}})^2$

We obtain an equivalent equation of third degree in $θ$:

\begin{equation}

θ^3−8θ^2+8(3−2α)θ−16(1−α)=0.   (2.13)

\end{equation}

Using Cardan’s formula and taking advantage of MAPLE procedures, we get a formula for the $θ$, where we have three solutions which can be pure real, pure imaginary or complex depending on the value of the Poisson ratio ν. The first root is given by

$θ_{1}=\frac{2}{3}\sqrt[3]{(-1745α +3\sqrt{33 − 186α + 321α^2 − 192α^3})}$

$-\frac{3}{2}\frac{(\frac{8}{9}-
\frac{16}{3}α)}{\sqrt[3]{(−17+45α+3\sqrt{33−186α+321α^2−192α^3)

}}}+\frac{8}{3}$

This root then describes the pure Rayleigh surface wave $θ_{R} = θ_{1}$. The phase velocity c of Rayleigh waves is obtained as

$c=\frac{c_{t}}{\sqrt{θ_{R}}}$

6.0 Numerical modelling and result analysis

The software (MATLAB) is used in this work for analyses.

Here, the MATLAB codes have been presented. The frequency response function is calculated for a homogeneous soil profile. The frequency response functions can be found for 3 cases of the soil profiles. They are: uniform undamped soil layer over rigid bedrock, uniform damped soil layer over rigid bedrock and uniform damped soil layer over elastic bedrock. The various inputs include : thickness of the soil layer (in m) [H] ,mass density of the soil and the bedrock (in kg/m3), shear wave velocity of the soil and bedrock (in m/s), Vs = [Vs_soil Vs_rock], hysteretic material damping of the soil and the bedrock, Kips per square inch [KSI], sampling frequency for the analysis [Fs] and number of frequencies for the Fast Fourier Transform (FFT) . The values are first initialized and the frequency vector is created. The Frequency Response Functions (FRE) matrix is initialized for speed. The speed is pre allocated and the transfer function is written. The wave propagation is studied here. The displacement wave field is calculated here in a layered medium using elastic wave equation. The displacement response is calculated at the interfaces of an elastic multilayered medium to a vertically propagating unitary Shear velocity or SH elastic wave. The calculation of the displacement response in a layered soil profile is done. To find the transfer functions between the interfaces can be found by dividing the total displacement response which is the sum of the upward and downward values on the corresponding interfaces.

The Phase velocity vs Frequency, the shear wave velocity vs depth, and the vertical particle displacement profile vs penetration depth is analyzed.

Shear wave velocity vs Depth MATLAB coding

MATLAB Code:

Matlab Code :

```
x = [0 1 2 3 4 5 6 7 8 9 10]
y = [200 180 150 210 500 520 500 550 600 610 620]

plot(x,y)


xlabel('Depth')
ylabel('Shear wave velocity')

x = [0 1 2 3 4 5 6 7 8 9 10 ]


y = [4 4.5 5 5.3 4.2 3.8 2.8 1.5 1 0.1 0 ]


plot(x,y)
xlabel('Penetration depth')

ylabel('Vertical particle displacement distribution')

title('Vertical particle displacement profile')
x = [ 0 1 2 3 4 5 6 7 8 ]

y = [ 7 6 5 4 2 3 4 5 6]

plot(x,y)

ylim([0 7])

xlabel('frequency')

ylabel('Phase Velocity')

title('Dispersion Curve')

xlabel('frequency')

ylabel('Phase Velocity')
```

title('Dispersion Curve')

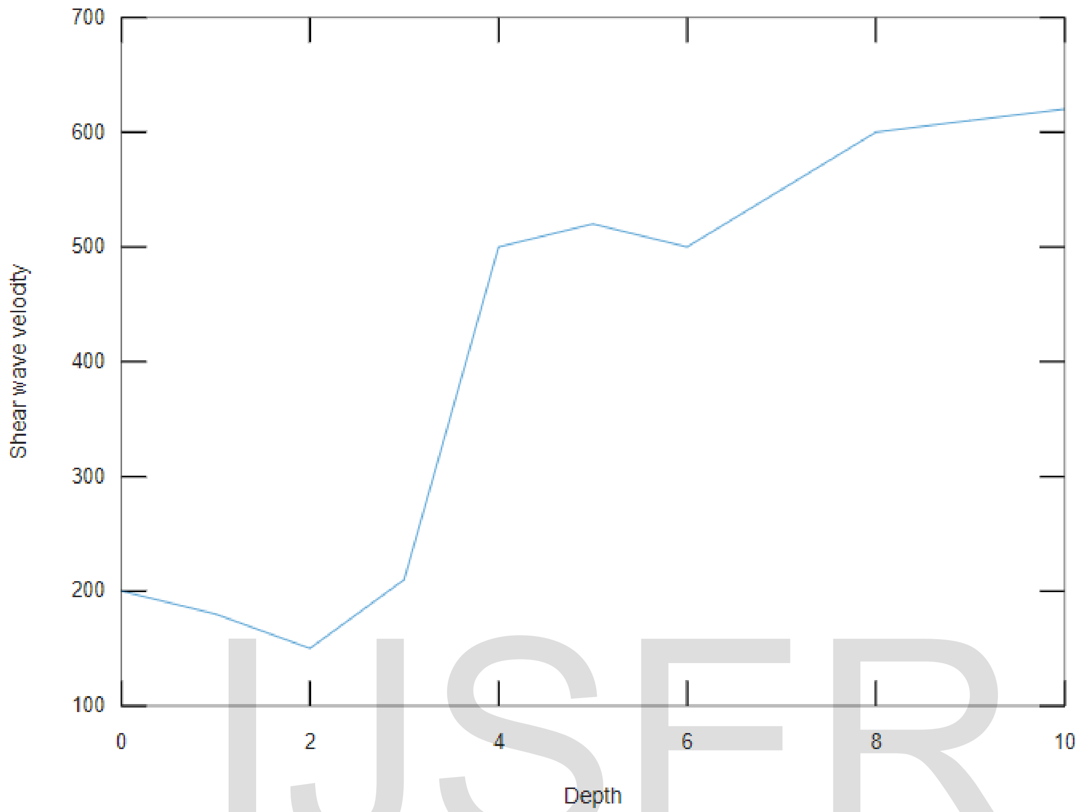

Figure 1

The Figure 1 shows the variations of the shear wave velocity versus depth. It can be seen that the variation is in the form of steps. As can be seen from this figure, the model displays generally an increasing trend as existed usually in nature. It indicates two layers geological medium including a soft soil layer as a top soil with the velocity values ranging almost from 150 to 200m/s overlaying a stiffer layer with the velocity magnitudes within 500 to 600m/s.

The shear wave velocity can be calculated mathematically as: V= Square root of √G/ρ

The velocity (ν) of a shear wave is equal to the square root of the ratio of shear modulus (G), a constant of the medium, to density (ρ) of the medium, ν = Square root of √G/ρ. Both shear (transverse) and compressional (longitudinal) waves are transmitted in bulk matter.

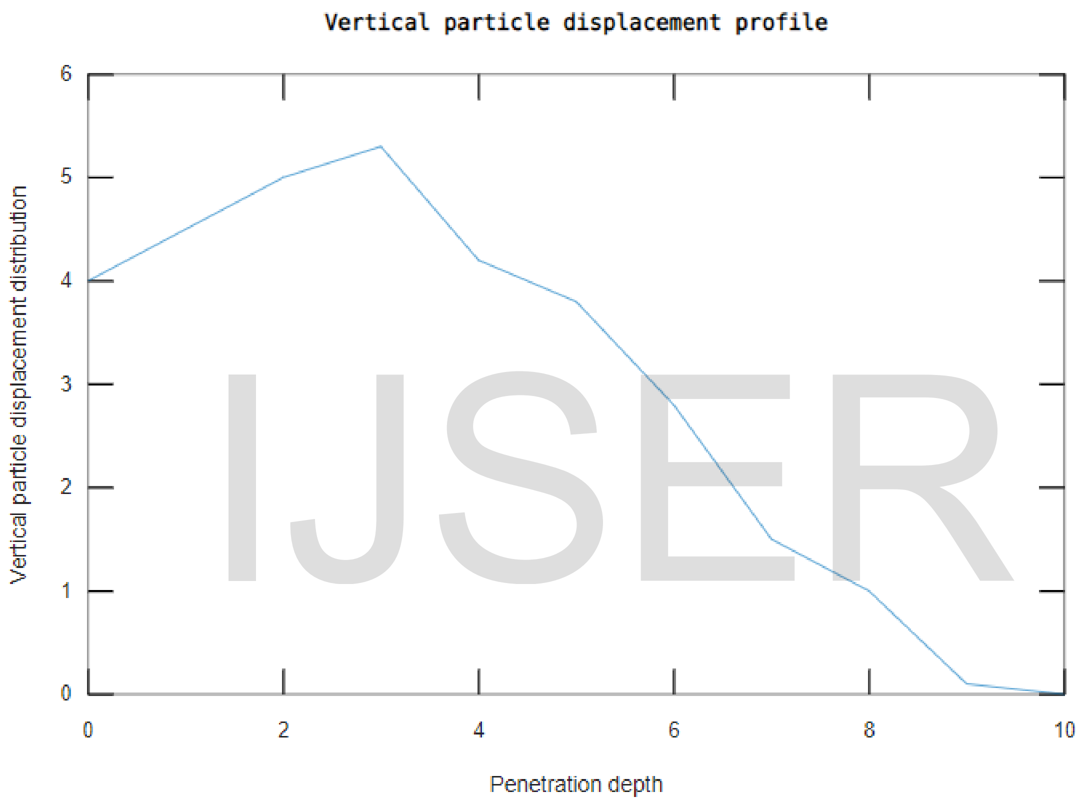

Figure 2

The Figure 2 shows the vertical particle displacement profile which is the variation of vertical particle displacement distribution with respect to the penetration depth. It can be seen that initially the vertical particle displacement rises till a penetration depth and then falls as the penetration depth rises.

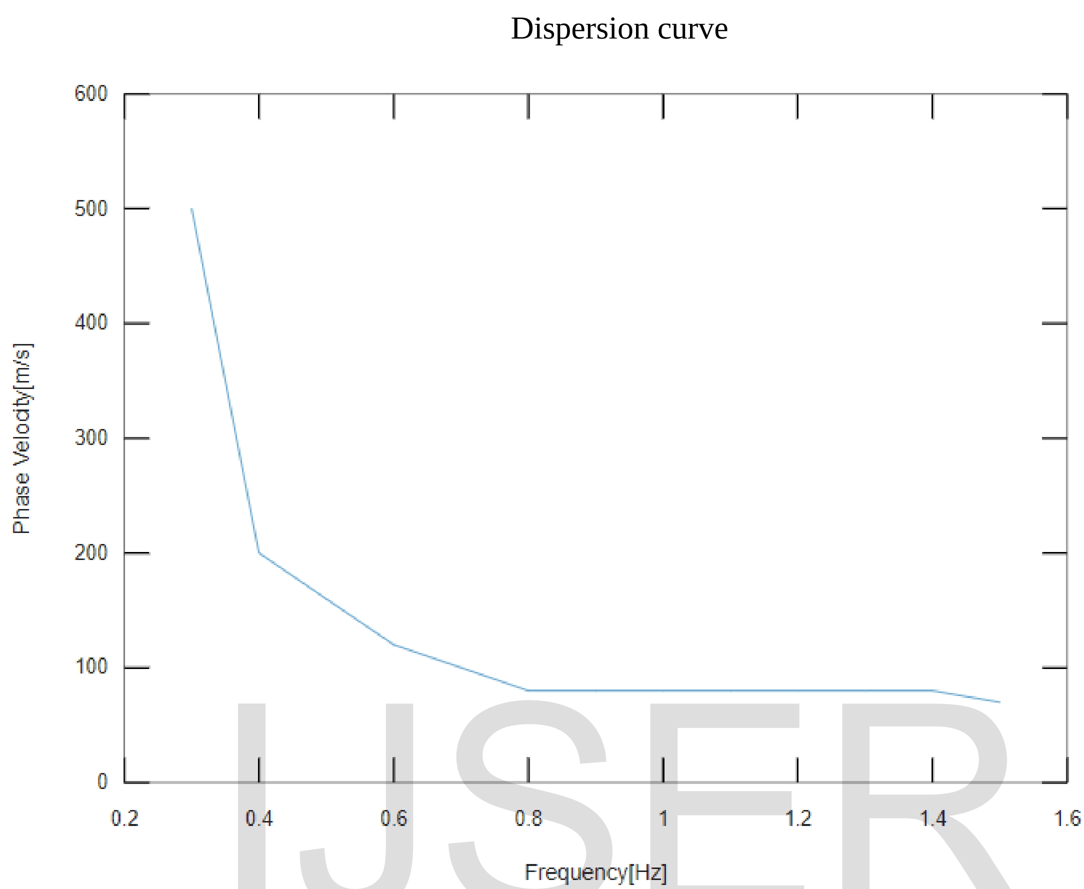

Figure 3

The Figure 3 shows the variations of phase velocity for different frequency components of Rayleigh waves train, which is called as a dispersion curve of fundamental mode of Rayleigh wave. It is a very important outcome so that can influence directly the accuracy of resulting shear waves velocities. This curve presents that each frequency component travels along the surface ground with a specific velocity and value and penetrates to a determined depth depending to its wavelength. It is worth mentioning that the lower frequency components (i.e., longer wavelengths) can penetrates to the larger depth values and reveal useful information on the Shear wave velocity magnitudes of deeper geological formations.

However, the higher frequency components (shorter wavelengths) characterize the pattern of S-wave velocity distribution in the shadow soil and rock layers.

It should be noted that as long as an extensive frequency band can be extracted from collected Rayleigh waves from the real in-situ tests, the obtained S-wave velocity model from this technique

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contains a wide range of geomaterials in various depths.

Therefore, depending to the actual applications in industrial and research studies, the designing for acquisition of Rayleigh waves is performed so that collect the required frequency range in the light of project's objectives. The phase velocity is given in terms of the wavelength λ and time period T as v p = λ T.

7.0 Conclusion and recommendation

Conclusion:

Hence, the modelling of the surface (Rayleigh) waves data was performed using MATLAB. The outcomes from the program result in the variations have been presented. The Rayleigh waves are used to extract the shear wave velocity in the soil. It can be seen that reliable near surface S wave velocities can be obtained from data for Rayleigh wave dispersion. Shear wave velocity is the property which dominates (out of Shear wave velocity, P – wave velocity, thickness and density) in the layered earth model in case of the fundamental mode for the Rayleigh dispersion data at high frequency. The weighted solution can be solved using the Levenberg–Marquardt algorithm (L-M) method and singular value decomposition technique (SVD) using iterative solution. The borehole Shear wave velocity measurement can be used to verify the inverse result.

Recommendation:

In this study, important relationships were found between the shear wave's velocity and other geo-technical parameters. This study recommends this method for future projects. This method can also be used to obtain preliminary results on soil characters.

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Appendix

Phase velocity: The phase velocity of a wave is the rate at which the wave propagates in some medium. The phase velocity is given in terms of the wavelength λ and time period T as v p = λ T.

Damping factor: is mathematically modelled as a force with magnitude proportional to that of the velocity of the object but opposite in direction to it Uniform damping:

Elastic multilayered medium: Typically, an elastic multi-layered medium consists of two or more structural layer components bonded to varying degrees.

Borehole: Borehole are basically used for taking soil sample from different places in the proposed site for construction.

RHO: The reinforcement ratio.

P-Wave: Is one of the two main types of elastic body waves, called seismic waves in seismology.

SASW: Spectral analysis of surface waves (SASW) is a non-destructive method of testing the shear wave velocity profile of soil and rock.

In-situ tests: In situ testing is a division of field testing corresponding to the cases where the ground is tested in-place by instruments that are inserted in or penetrate the ground.

Uniform Undamped Soil on Elastic Rock: Maintaining equilibrium and compatibility of displacement at the boundary.

SH waves: SH waves are decoupled from P and Shear velocity waves, the transmitted and reflected waves will also be SH waves. Recall that displacement corresponding to SH waves satisfies the wave equation.

References

[1] Wu, Dunshi, et al. "A MATLAB Package for Calculating Partial Derivatives of Surface-Wave Dispersion Curves by a Reduced Delta Matrix Method." Applied Sciences 9.23 (2019): 5214.

[2] Xia, Jianghai, Richard D. Miller, and Choon B. Park. "Estimation of near-surface shear-wave velocity by inversion of Rayleigh waves." Geophysics 64.3(1999): 691-700.

[3] Li, Peiming, et al. "Near-surface shear-wave velocity estimation based on surface-wave inversion." The Leading Edge 35.11 (2016): 940-945

[4] Lefeuvre, F., P. Desegaulx, and M. L. Baratin. "Shear-wave velocity estimation." SEG Technical Program Expanded Abstracts 1993.Society of Exploration Geophysicians, 1993. 711-714.

[5] Li, Yonghua, et al. "S-wave velocity structure of north eastern China from joint inversion of Rayleigh wave phase and group velocities." Geophysical Journal International 190.1 (2012): 105-11